

A TRANSIENT-EVENT MODEL FOR EEG POWER SPECTRA

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This report describes results obtained by modeling the EEG as a collection of overlapping wavelets. Using only linear theory, it provides a basis for understanding the presence of peak frequencies, as well as harmonics and sub-harmonics, in the EEG. It also predicts observed resonance phenomena, in terms of a linear model.

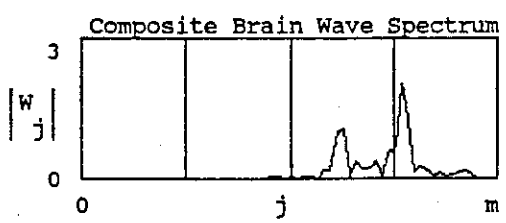
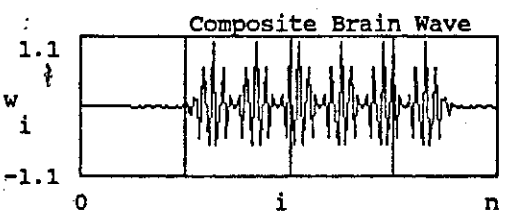
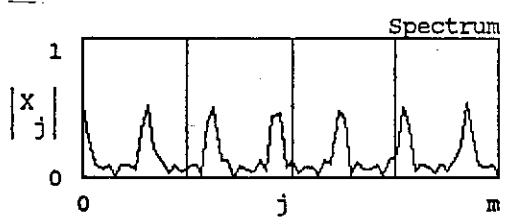
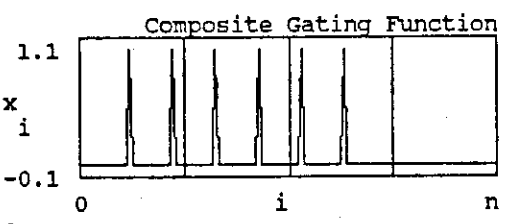
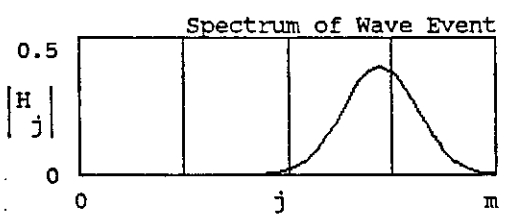
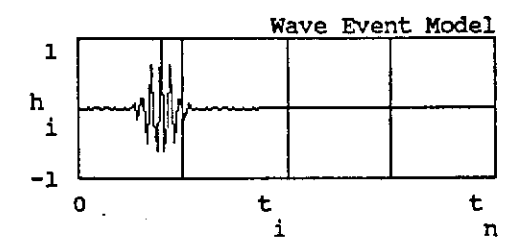
Current modeling efforts generally lean toward nonlinear theory, limit cycles, and wave equations. While providing a possible basis for understanding dominant EEG rhythms, simpler approaches should be sought. It is possible to build an alternative model, which appeals only to the notion of linear superposition, yet still explains key phenomena.

The model is based on the presence of synchronizing activity, which stimulates, or evokes, transient cortical events. Based on the nature of the transient wavelets, and the characteristics of the synchronizing activity, the model produces frequency peaks and harmonics, which replicate observed spectra. In addition, the model provides a uniform approach to understanding sensory EEG "driving", without using nonlinear theory.

It is found that this approach provides a physiologically intuitive rationale for the presence of harmonic and sub-harmonic activity in the EEG, as well as evoked resonance phenomena resulting from sensory stimulation.

Nunez, P. "Electric Fields of the Brain," Oxford, 1981.

Collura, T. F., "Real-Time Filtering of Transient Visual Evoked Potentials," 30th ACEMB.



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A MODEL FOR TRANSIENT BRAIN PROCESSES UNDERLYING EEG POWER SPECTRA

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The origin and nature of brain waves as measured from the human scalp has been a topic of ongoing research. Developments by Nunez and others have provided a rigorous foundation for attributing EEG signals to cortical sources; subsequent work by Gevins et. al. has demonstrated the use of the inverse Laplace Transform as a means of recovering the hypothesized source potential distributions.

A model is needed, however, which develops the specific observed EEG signal characteristics, from a morphological point of view. Whereas earlier work establishes the theory for a physical layer of EEG genesis, the next higher level would deal with particular signal characteristics relative to physiology and phenomenology, to explain why we in fact see precisely what we see.

The following model is presented as a candidate for this purpose. It draws from theory and experience in the area of Event Related Potentials in general, and Sensory Evoked Potentials in particular. It addresses observations relative to EEG power spectra, their time behavior and stability, and their relationship to externally and internally synchronized events. (Collura, 1978)

The model builds up the eeg as a sequence of transients. These transient waveforms are not unlike classical event-related-potentials (ERPS), in that they are cortical brain potentials of finite duration, consisting of a series of wavelike "peaks" and "valleys". The major difference is that these wavelets are presumed to be time locked to intrinsic sub- or trans-cortical brain activity, not to external stimuli or events.

The following development is implemented using an engineering spread-sheet (MathCAD by MathSoft), which ensures that all expressions are accurate and correctly plotted. To accomodate this format, we first define a set of constants and range variables, which will form the foundation of the analysis.

Since the spectral analysis (fft) is based on a power-of-two, the calculations are based on choosing, first the power of two, commonly "nu":

$$nu \equiv 7$$

This determines the number of points in the data arrays, in both the time domain, and in the frequency domain for the Fast Fourier Transform. The array sizes, and their associated 'range variables', are set up as follows:

$$\begin{array}{ll}
 n := 2^{\text{nu}} - 1 & m := 2^{\text{nu}-1} \\
 n = 127 & i := 0 \dots n \\
 \text{(Time Domain)} & \text{(Frequency Domain)}
 \end{array}$$

Next, we choose a 'sampling interval', which is the time interval between data samples. This choice then dictates the length of the epoch, plus the spectral resolution and spectral bandwidth of the frequency spectrum.

The interval between samples is chosen to be $\gamma \equiv 0.008$ seconds,

which is equivalent to a sampling rate of $\frac{1}{\gamma} = 125$ samples per second.

We then set up the time axis: $t_i := i \cdot \gamma$

And determine the epoch length: $T := \frac{t_n}{n} = 1.016$ sec.

In the frequency domain, the axis is defined: $f_j := \frac{j}{T}$

Which determines the spectral resolution: $f_1 = 0.984$ Hz

and bandwidth (maximum frequency): $f_m = 62.992$ Hz

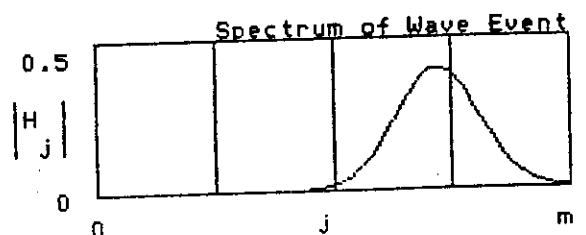
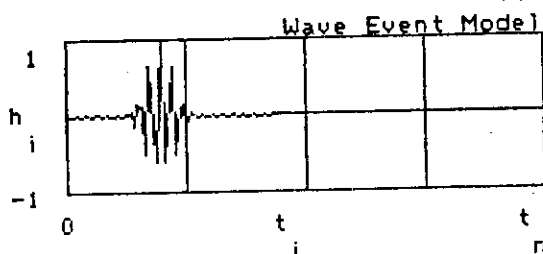
To proceed with the model, example waveforms are computed via equations and tables which appear in connection with the graphic output.

First, a transient wavelet is defined, and its transform is computed:

$$h_i := \cos \left[2 \cdot \pi \cdot f_0 \cdot t_i \right] \cdot e^{- \left[2 \cdot \pi \cdot \left[\frac{t_i - \gamma_0}{T \cdot \gamma_d} \right]^2 \right]}$$

$f_0 \equiv 45$ "Center frequency"
 $\gamma_d \equiv 0.25$ "Duration"
 $\gamma_0 \equiv 0.2$ "Latency"

$H := \text{fft}(h)$



Note that, although the spectral energy of the wave event peaks at the 'carrier' frequency of 50 Hz, the signal is broadband in nature, and has finite energy over half of the measurable frequency range. This is due to the amplitude modulation implicit in the finite duration of the wavelet. The shorter the duration of the wavelet, the more frequencies it contains, thus, the broader the frequency spectrum.

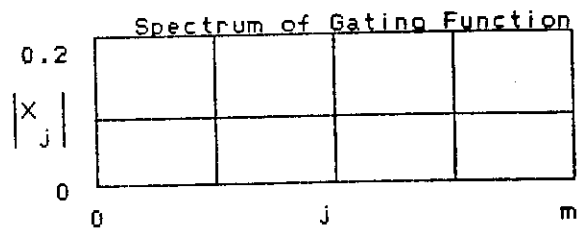
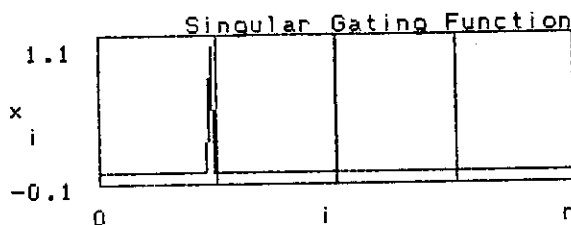
Next, a gating function is defined, which will time-lock one or more occurrences of the wave event. We do this by setting up a function, x , which is zero everywhere except at the instant when an event will occur:

$$x := 0 \quad \text{for all } i, \text{ except at } i$$

$$p := 30, \text{ where } x := 1$$

$$p$$

$$X := \text{fft}(x)$$



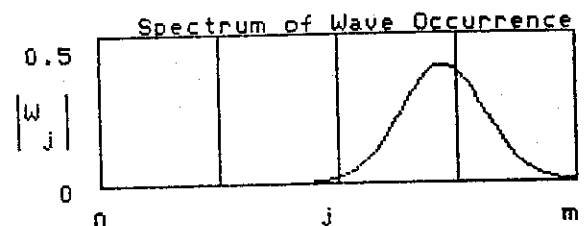
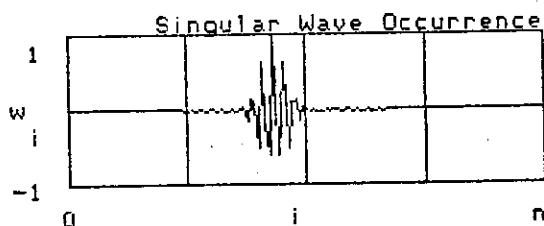
Since this gating function is a single impulse, it possesses all frequencies, and its spectrum is totally broadband. When it is used to synchronize the wave event, therefore, it will introduce all frequencies present in the wave. This results directly from the frequency multiplication property of the Fourier Transform:

(The MathCAD primitive $\delta(t-\gamma)$ is the heaviside (step) function)

$$w := \delta(i - p) \cdot h$$

$$i \quad i-p$$

$$W := \text{fft}(w)$$



This illustrates that if the eeg contains a relatively short burst of a characteristic frequency, the resultant frequency spectrum will contain a broad energy peak, which is not at all limited to the characteristic frequency itself. The breadth of this frequency band will be inversely related to the duration of the pulse; a briefer pulse will result in a broader spectrum, while longer burst creates more narrowband activity.

Next, we introduce additional occurrences of gating pulses. To do this, we define a table of coordinates representing the instants at which pulses occur. As before, the gating function is zero for all other times.

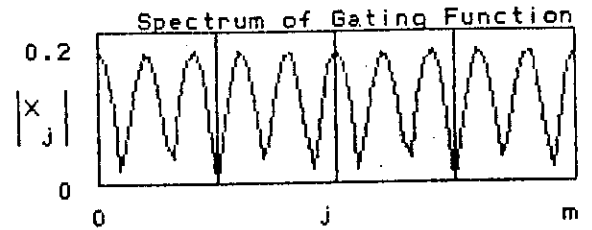
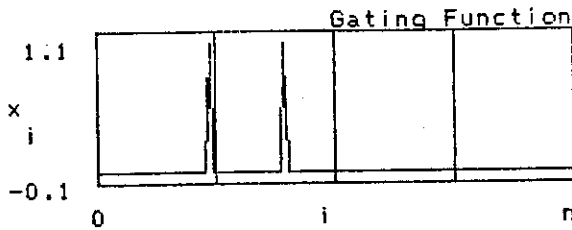
```

x := 0      ip := 1 .. 2      p :=
i
          x := 1
          p
          ip

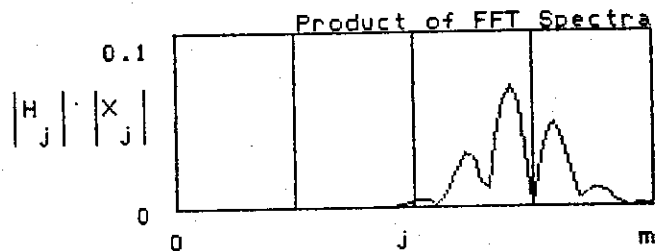
```

ip
30
50

X := fft(x)



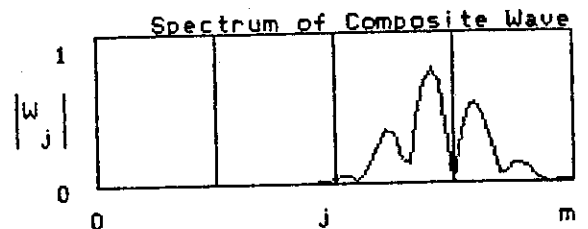
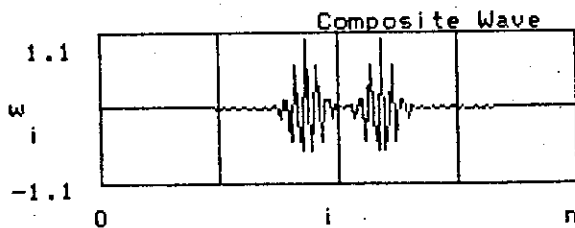
As an illustration of the frequency multiplication property, we will first compute, in the frequency domain, the product of the wave event and the gating function. This demonstrates the phenomenon by which the gating function is seen to sample, or 'window in', selected frequencies from the wave event.



Next, we compute the composite wave through the use of convolution, which mathematically superimposes occurrences of the wave event, subject to the instants defined by the gating function. The spectrum of this composite wave, which is computed from the Fourier Transform, is found to agree, via a scaling factor, with the spectrum computed via frequency domain multiplication. This is a general result, which will always hold under the conditions of this model.

$$w_i := \sum_{ip} \delta[i - p_{ip}] \cdot h_{i-p_{ip}}$$

W := fft(w)



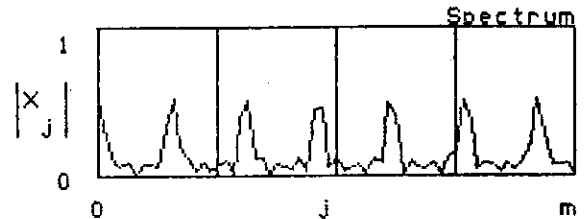
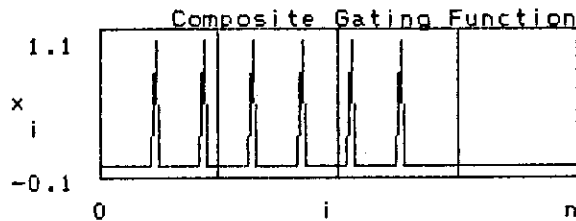
To further develop the implications of this model, we look at more complex gating functions, which contain more impulses, and with various temporal characteristics. At one extreme, we can have a perfectly regular, though finite, train of impulses, providing a synchronizing 'clock':

```

ip := 1 ..6      p :=
                ip
                15
                28
                41
                54
                67
                80
x := 0
i
x := 1
p
ip

```

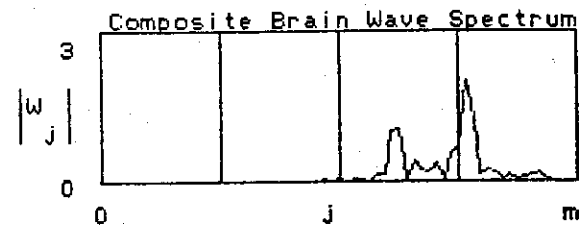
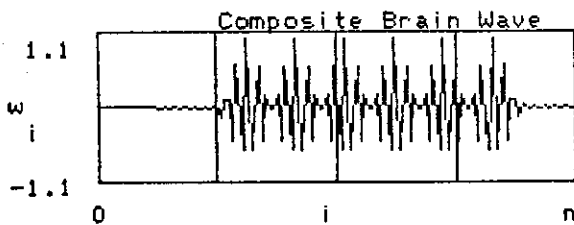
```
X := fft(x)
```



The composite wave now consists of a series of bursts, paced at regular intervals. The spectrum of this sequence therefore consists of a set of peaks whose energy is dictated largely by the shape of the wave event, while the location and spacing of the peaks is dictated by the statistics of the underlying composite gating function:.

$$w_i := \sum_{ip} \phi [i - p] h_{i-p}$$

```
W := fft(w)
```



If the pulses contained in the gating function are indeed periodic and regular, the frequency spectrum will also contain regularly spaced peaks, whose spacing is inversely proportional to the delay between gating pulses. If noise in the form of jitter is introduced, the gating spectrum becomes less structured, and does not introduce energy at well-defined locations:

ip := 1 .. 6

p :=

ip
18
28
48
62
69
95

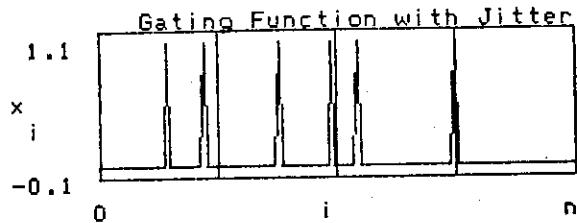
x := 0

i

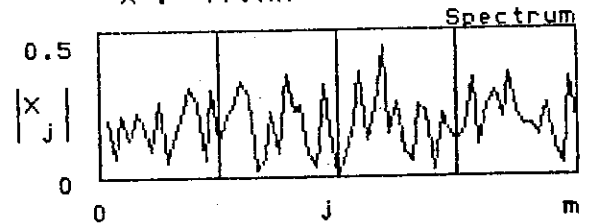
x := 1

p

ip

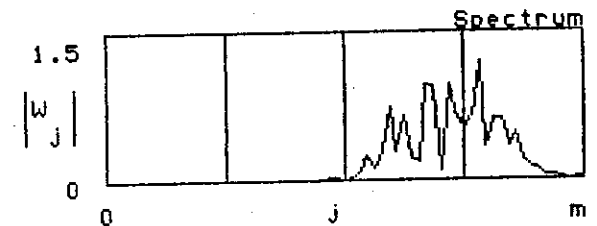
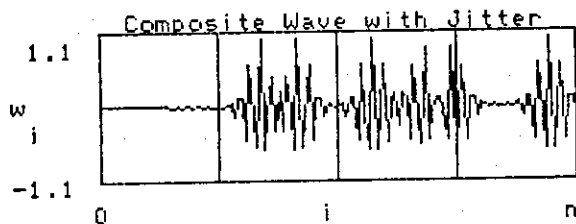


X := fft(x)



$$w_i := \sum_{ip} \Phi \left[i - p_{ip} \right] h_{i-p_{ip}}$$

W := fft(w)



Under conditions of very unstable periodicity in the gating function, the resultant spectrum contains a fine-structure which is related only in a statistical way to the synchronization of the wave events. In other words, if one were to compute the average of many such spectra from a single subject, the peaks and valleys would cancel out, resulting in an average spectrum not unlike that of a singular wave event. This implies that, to the extent that fine spectral peaks are found to be stable in frequency, they imply the presence of underlying synchrony.

This suggests that, to the extent that fine spectral detail is found to be stable, and persists in the mean, that implies the presence of relative synchrony in the gating process.

In addition, the model suggests that whenever one such stable frequency peak is identified, it is reasonable to look for other peaks, with some regular spacing in the frequency domain. Depending on the shape of the wave event spectrum which determines the 'weighting factor' for each frequency, certain frequencies may be enhanced or suppressed, so that one might find particular harmonics in strength, but not others.

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